Abstract—Hybrid electric vehicles (HEV), plug-in HEV (PHEV) need an energy management system (EMS) to ensure good fuel economy while maintaining battery state-of-charge (SOC) within a safe range. The EMS is in charge of the power split decision between the engine and the electrical motor. For a PHEV, the optimal power split strategy will depend on the driving cycle, initial SOC, and trip length. Heavy computation and accurate knowledge of the future trip are required to find the optimal power split control and this represents a significant difficulty for the development of an EMS. The aim of this paper is to propose a genetic algorithm (GA) that optimizes the power split control parameters for a given driving cycle in a relatively short computation time, thus, overcoming the problem of heavy computation. The methodology consists in 1) defining the control laws and their associated control parameters based on the observation of optimality obtained by dynamic programming; and 2) developing a GA that will be able to compute the near-optimal values of these parameters in a short time and for a given driving cycle. It is demonstrated that the GA provides short computational burden and near-optimality for a wide variety of driving cycles. It then offers a promising tool for a future real-time implementation.

Index Terms—Energy management system (EMS), genetic algorithm (GA), plug-in hybrid electric vehicles (PHEV), power split strategy, three-wheel electric vehicle (EV).

I. INTRODUCTION

Since the end of the XXth century, the automotive industry is facing the challenge of reducing fuel use and emissions caused by transportation. A promising solution is, of course, vehicle electrification. Since the release of the Toyota Prius, which is the first modern commercial hybrid electric vehicle, many car manufacturers have launched hybrid electric vehicles (HEV), plug-in hybrid electric vehicles (PHEV), and pure electric vehicles (EVs). EVs only have one electrical motor (EM) and no internal combustion engine (ICE). As a consequence, they do not consume fuel while running. However, the main restriction toward a wide-spread use of this technology is the high cost of the battery for an acceptable driving range.

Hybrid technology, used in both HEVs and PHEVs, overcomes these drawbacks by combining an ICE and an EM in the powertrain. This allows a significant increase in the driving range and a better fuel economy compared to conventional vehicles. Therefore, PHEVs are considered as long-term technologies by some authors [1], [2] as they promise a better fuel economy compared to HEV. Hybrid powertrain can be made with various configurations: parallel, series, mild hybrid, series-parallel. The parallel hybrid configuration has the advantage of a downsized EM and drive compared to the series configuration. In this paper, only the parallel hybrid configuration with one EM and one ICE will be studied. PHEVs include a high-level controller, named energy management system (EMS), which minimizes the vehicle fuel consumption and manages the battery state-of-charge (SOC) by accurately assigning the required power level to be supplied by the EM and the ICE. In the parallel hybrid configuration, a key control parameter is the percentage of the demanded power to be supplied by the ICE, hereafter called the power split ratio (PSR). The controller must then establish if the ICE should be off (PSR of 0%), if the required power should be split between the EM and the ICE (PSR between 0% and 100%) or if the ICE should be overdriven (PSR over 100%), with the EM operated in generator mode.

Control strategies are usually classified into rule-based and optimization-based categories for both HEV [3] and PHEV [4]. The rule-based strategies are based on deterministic [5], [6] or fuzzy rules [7]–[9]. Generally, these rule-based strategies make the PHEV run in pure electric mode like an EV during the first part of the trip, and then impose the use of the ICE to perform charge sustaining (CS) operation like an HEV. Rule-based strategies are widely used in industry because of their simplicity and reliability; however, they exhibit a lack of optimality because they only rely on the current driving conditions [4]. Optimization-based strategies use a mathematical approach to determine the optimal PSR. Global optimization has been particularly investigated as it is able to find the optimal control sequence of the two motors over a complete predefined driving cycle. A very common global optimization tool is
dynamic programming (DP) [10], [11]. Interestingly, some authors have applied DP to a PHEV and have concluded that the vehicle should not work in charge sustaining during any portion of a given trip of any distance. The optimal control of the PHEV power split ratio will depend on the speed profile and range of a given trip, which represents an additional complexity compared to HEVs. Unfortunately, DP needs lengthy computation, cannot provide control laws that can be easily used in real-time and furthermore, can only be performed if the entire speed profile is previously known. These drawbacks prevent from using it directly in real-time.

To overcome these difficulties, several alternate techniques are proposed either for HEVs or PHEVs. At first, real-time control laws can be inferred directly from observation of the DP optimal results [12], [13]. A similar approach consists in using the DP results into machine learning in order to build efficient real-time control laws [14]–[16]. Furthermore, gradient-based analytical approach [17] or meta-heuristic optimization like genetic algorithms (GA) [18], [19], DIRECT algorithm [20], simulated annealing [21] and particle swarm optimization [22] are used as ways to compute optimal real-time control laws while reducing the computational burden implied by DP. However, these techniques perform global optimization and consequently the obtained control laws are optimal for one particular driving cycle.

The Pontryagin’s minimum principle (PMP) is also used to perform global optimization and allows proving, under mild assumptions, that global optimality can be achieved by using an equivalent consumption minimization strategy (ECMS) with a constant equivalence factor [23], [24]. This equivalence factor appears to be closely related to the costate of the PMP problem. The ECMS has the tremendous advantage of being easily implementable in real-time but its optimal equivalence factor depends on the driving cycle.

A common point of the methods presented here is the need of the a priori knowledge of the future driving cycle. There is an inherent difficulty in prediction of the future speed profile, as it is subject to much uncertainty. Assuming that the future driving cycle can be forecasted more or less accurately, an EMS should ideally be able to compute optimal power split control parameters that minimize the fuel consumption on the predicted driving cycle, in a short amount of time and computation requirement.

In this paper, it is proposed to apply DP offline to obtain the optimal ICE torque control on a set of normalized predefined driving cycles that represent different driving conditions. By accurate observation of these offline results, two power split control laws are determined. In particular, a speed dependent torque threshold is identified for the working mode transition control law. The optimal threshold appears to vary with driving conditions, initial SOC and environmental conditions. This kind of control law has already been identified in the literature [12], [13] but no algorithm has been proposed to compute the optimal threshold rapidly and for a wide variety of driving cycles. This is essential since the DP requires heavy computation and driving conditions can change drastically. In this paper, the threshold of the transition control law is defined mathematically by three parameters $a$, $b$ and $c$. A GA is then proposed as a fast optimization algorithm that computes near-optimal $a$, $b$ and $c$ parameters for several predefined driving cycles regarding the fuel minimization objective.

The paper is divided into 4 sections. Section II will present the generalized-optimized form of the power split control law proposed after observation of the optimal DP results. In particular, this section will describe the working mode transition threshold mathematically, by use of the parameters $a$, $b$ and $c$. Section III will present the GA and its use to optimize $a$, $b$ and $c$. Section IV will discuss the optimality of the proposed GA by comparing its performance to DP and a standard rule-based strategy.

II. GENERALIZED FORM OF THE POWER SPLIT CONTROL LAW

A. DP Algorithm and PHEV Architecture

Dynamic programming is an optimization algorithm that is particularly well suited to non-linear systems [25], such as the powertrain of a HEV or PHEV. As previously discussed, DP has been used extensively in past publications. Here, it will be used to infer a generalized form of the power split control law. The architecture of the parallel hybrid powertrain is described in detail in the following subsection.

B. Parallel Hybrid Vehicle Architecture Considered

The vehicle described in this paper is a three-wheel EV that already exists in its conventional form (with only an ICE). In order to reduce its fuel consumption without sacrificing its driving range, it has been proposed to develop a plug-in hybrid form of this vehicle. The challenge is therefore to modify the complete powertrain without altering the vehicle design too much. The constraints of space are a major concern and thus greatly influenced the choices made for the powertrain architecture. A schematic of the vehicle is provided in Fig. 1.

A parallel topology has been chosen because a preliminary study has proved that this topology would take less space than
a series architecture. The specific geometry of the vehicle requires a primary and secondary drive for the disposition of the powertrain components. The powertrain is composed of an ICE and an EM, thus providing suitable mechanical power to the rear wheel through a set of gears and belt. The EM can work either in motoring or regenerating mode. The required power \( P_{\text{req}} \), which is the demanded wheel mechanical power (see (2)), can be negative in case of braking. The corresponding energy flow chart is given in Fig. 2 where \( P_{\text{ICE}} \) and \( P_{\text{EM}} \) are ICE and EM output power respectively. As illustrated, ICE power is assumed to be positive only (from ICE to gearbox), or zero when the clutch is decoupled.

The various gears that compose the mechanical transmission are defined by their respective ratio and efficiency. The gearbox allows fixed gear selection among 6 different gear ratios. The selected gear ratio number is denoted as \( k \). The different notations are defined in Table VII given in Appendix A. The complete vehicle model was developed in a previous work [26], which contains more detailed information.

Based on this drivetrain configuration, it is possible to relate rotational speeds of ICE \( N_{\text{ICE}} \), EM \( N_{\text{EM}} \) and wheel \( N_w \) as in

\[
i_{\text{fin}} N_w = \frac{N_{\text{ICE}}}{i_{\text{pr}} i_{\text{sec}} i_{\text{gb}} (k)} = \frac{N_{\text{EM}}}{i_{\text{mg}}} \tag{1}
\]

The ICE torque \( T_{\text{ICE}} \), EM torque \( T_{\text{EM}} \) and total demanded torque \( T_{\text{req}} \) follow (2). This relation is dependent upon the gear efficiencies and gear ratio, which numerical values are defined in Table VI of Appendix A. The total demanded torque \( T_{\text{req}} \) is set by the driver and the EMS will choose \( T_{\text{ICE}} \) and \( T_{\text{EM}} \) with respect to the architecture constraint (2),

\[
\frac{T_{\text{req}}}{i_{\text{fin}}} \left[ \frac{1 - sg(T_{\text{req}})}{2} \eta_{\text{fin}} + \frac{1 + sg(T_{\text{req}})}{2} \right] = T_{\text{EM}} i_{\text{mg}} \left[ \frac{1 + sg(T_{\text{EM}})}{2} \eta_{\text{mg}} + \frac{1 - sg(T_{\text{EM}})}{2} \eta_{\text{mg}} \right] + T_{\text{ICE}} \eta_{\text{pr}} \eta_{\text{sec}} \eta_{\text{gb}} i_{\text{pr}} i_{\text{sec}} i_{\text{gb}} (k) \tag{2}
\]

where \( i_{\text{fin}}, i_{\text{mg}}, i_{\text{pr}}, i_{\text{sec}} \) and \( i_{\text{gb}} \) are mechanical gear ratios and \( \eta_{\text{fin}}, \eta_{\text{mg}}, \eta_{\text{pr}}, \eta_{\text{sec}} \) and \( \eta_{\text{gb}} \) are the gear efficiencies. The signum function \( (sg) \) is used to take the power sign into account. The power is either divided or multiplied by the gear efficiency depending on whether it is positive or negative.

Optimization of the fuel consumption requires that all the equipment of the vehicle where energy is dissipated be modeled accurately. In the numerical model of the PHEV, which relate vehicle speed to the required torque. The ICE fuel aerodynamic drag and rolling resistance are key parameters efficiency is also modeled using the consumption map shown in Fig. 3, relating ICE speed and torque with instantaneous fuel consumption. The electrical powertrain is composed of a Li-Ion battery pack which provides power to a voltage source IGBT inverter. The inverter drives a permanent magnet synchronous motor (PMSM). All the components of the electrical powertrain are modeled using common mathematical formulations [26]. The specifications of both ICE and electrical powertrains are also listed in Table I.

**Fig. 2.** Energy flow chart of the vehicle.

**Fig. 3.** ICE instantaneous fuel consumption (g/h) for the vehicle considered in this study.
C. Power Split Ratio From Dynamic Programming Algorithm

The ICE torque, $T_{\text{ICE}}$, and the gear ratio number $k$ are chosen as the control variables of the DP algorithm. The DP algorithm computes the optimal control variables at each time step of the driving cycle to minimize the fuel consumption. The other system variables values are inferred using the mathematical model. The dynamic system of DP is the battery SOC. If the speed cycle is divided into $N$ samples and $t_i$ is the time at the $i$th sample then the SOC at time $t_i$ and the SOC at time $t_{i+1}$ follow (3). The SOC dynamic behavior depends on $T_{\text{ICE}}$, $k$, the required torque $T_{\text{req}}$ and the wheel speed $N_w$ (or, by extension, the vehicle speed) given by

$$\text{SOC}(t_{i+1}) = \text{SOC}(t_i) + d\text{SOC}(T_{\text{ICE}}, k, T_{\text{req}}, N_w).$$

The cost function to be minimized is defined as the fuel consumption summed up over the entire trip. As it can be seen in (4), the instantaneous consumption $D$ (in liters/s) depends on the wheel speed $N_w$, ICE torque $T_{\text{ICE}}$ and gear ratio number $k$ but is not made dependent upon the required wheel torque $T_{\text{req}}$. The ICE torque, $T_{\text{ICE}}$, can be applied independently from the required wheel torque, $T_{\text{req}}$; the EM compensates in order to satisfy (2) (as long as the EM operating limits are not exceeded). The total fuel consumption (in liters), $J_{\text{tot}}$, is then:

$$J_{\text{tot}} = \sum_{i=1}^{N} D(T_{\text{ICE}}, k, N_w) \Delta t,$$

where $\Delta t$ is the time step duration (in seconds).

For a complete definition of the minimization problem, constraints on the control variables are added, so as to keep the ICE and electric motor (EM) within their respective torque and speed operating limits. Constraints on the SOC are also added. To avoid unsafe operation, Dynamic Programming (DP) solutions containing values of SOC outside the range 95–20% are rejected. Also, the initial SOC can be set arbitrarily between 30% and 95% and the final SOC is set to 30% in all cases. Several values of initial SOC will be explored in the paper.

The problem being defined, the SOC should be discretized in order to build a time-dependent SOC grid. Using the Bellman’s principle of optimality [25], the DP algorithm will be able to find the optimal, least expensive path from the initial SOC up to the final SOC for a given driving cycle.

D. Power Split Control Laws Inferred From DP Observation

The DP algorithm was run offline on 10 facility-specific driving cycles, proposed by Sierra Research [27]. The 10 driving cycles represent three roadway types (urban arterial, freeway and local) with various levels of service (LOS) that are basically levels of traffic and facilities: the lower the LOS (letter “A”), the higher the average speed. The different driving cycles are listed in Table II and their corresponding speed profiles are given in Appendix B. DP results were observed on each of the 10 driving cycles in order to find simple power-split control laws, presented hereafter, that can mimic the optimal behavior while being easily usable in a real-time controller.

As can be observed in Fig. 1, there is no clutch on the EM shaft. Hence, the vehicle has two available working modes: pure electric (the vehicle is only propelled by the EM) and hybrid (both ICE and EM provide the mechanical power). The EMS has two main control laws; one that rules the power split during hybrid mode and one that rules the transition between pure electric and hybrid modes.

The original method proposed in the paper for the establishment of the two optimal control laws based on the DP results is primarily illustrated with the case of ART LOS AB with initial SOC of 50%, for an ambient temperature of 20 °C and no wind. The corresponding speed profile is illustrated in Fig. 4 and the optimal SOC evolution, as determined by the DP optimal solution, is shown in Fig. 5. The battery follows a progressive discharge, avoiding charge sustaining operation. Furthermore, it should be noted that the SOC decreases even when the vehicle is stopped. This is because the battery also provides the electrical power to the vehicle electronic apparatus through a DC/DC converter.

At first, the proposed method for setting up the EMS control law ruling the power split ratio in hybrid mode consists in plotting the ICE load points obtained from the DP optimization. These are illustrated in Fig. 6. It can be observed that the DP algorithm makes the ICE work near its maximum efficiency line (solid curve). Consequently, it is proposed to build a control law which sets the ICE torque and gear ratio so that the power split in hybrid mode maximizes the ICE efficiency at any given

<table>
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<tr>
<th>Table II: Facility-Specific Driving Cycles</th>
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<tr>
<td>Name [Acronym]</td>
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<tr>
<td>Arterial LOS A-B [ART LOS AB]</td>
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<tr>
<td>Arterial LOS C-D [ART LOS CD]</td>
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<tr>
<td>Arterial LOS E-F [ART LOS EF]</td>
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<tr>
<td>Freeway high-speed [FW HS]</td>
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<td>Freeway LOS A-C [FW LOS AC]</td>
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<td>Freeway LOS D [FW LOS D]</td>
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<td>Freeway LOS E [FW LOS E]</td>
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<td>Freeway LOS F [FW LOS F]</td>
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<tr>
<td>Freeway LOS G [FW LOS G]</td>
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<td>Local [LOCAL]</td>
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Fig. 4. ART LOS AB speed cycle.
time. This can easily be made in real-time using a 2-D control map that relates the ICE torque or gear ratio command with the vehicle speed and required torque. This control law appears to be simple and independent from the driving conditions or SOC. Yet simple, the proposed control law offers performances considered close enough to optimality, as will be demonstrated in Section IV.

Secondly, the proposed method for setting up the EMS control law ruling the transition between hybrid and pure electric mode consists in plotting the EM optimal load points $T_{EM}, N_{EM}$, calculated by DP. These are illustrated by the dots in Fig. 7. Moreover, another set of points is plotted, represented by the circles in Fig. 7 as the values of $T_{EM}$ and $N_{EM}$ that would be required from the EM if the vehicle was to run in pure electric mode during the whole driving cycle. Interpretation of Fig. 7 reveals some key features, which can be used to discriminate the hybrid mode from the pure electric mode. Wherever the circles and the dots match readily indicates that the optimal working mode chosen by the DP algorithm is pure electric. Otherwise, that is when the optimized dot points are not circled, the optimal working mode is hybrid, forcing the optimal EM load points to shift down in torque because of the positive torque contribution of the ICE.

Above 800 rpm, a discriminating line can be plotted on Fig. 7, above which black dots are not found: this is a threshold torque above which it is not optimal to run in pure electric mode. Moreover, at low speed (below the 800 rpm EM rotational speed) it can be seen that the DP algorithm forces the vehicle to operate in pure electric mode in order to prevent inefficient vehicle launch using the ICE and the clutch. Based on these considerations, it is proposed to build a control law that will compute a speed-dependent torque threshold and engage the pure-electric mode when the torque demand is below the threshold line, or enforce the hybrid mode for a torque demand above the threshold line. Such a torque threshold is observed here for the particular case of ART LOS AB with an initial SOC of 50%. Whether or not this kind of threshold can be generalized to every case remains to be investigated in the further sections.

E. Impact of the Driving Conditions

In order to confirm the existence of the transition threshold to a wide variety of cases, DP is applied on the 10 Facility-Specific Driving Cycles with initial and final SOC $=30\%$ at an ambient temperature of 20 °C and no wind. The results show that it is possible to infer a transition threshold in each case, as illustrated in Fig. 8. It can be concluded that the transition control law using the introduced torque threshold is usable in a wide variety of driving situations.

F. Impact of the Initial State of Charge

Contrarily to a HEV, a PHEV does not always operate in charge sustaining mode. Benefiting from the internal charger, a PHEV can use a fraction of its total embedded electrical energy throughout the trip. This is the case when the initial state of charge is above 30%. In this section, DP is applied for initial SOC values of 30%, 40%, and 50%. The three corresponding optimal battery discharge profiles are illustrated in Fig. 9. The corresponding torque thresholds obtained from DP load points inference are shown in Fig. 10.

Naturally, the possible contribution of the electrical power-train increases with the initial state of charge. Consequently,
Fig. 8. Transition thresholds for the different driving cycles with initial SOC of 30%, $T = 20\, ^\circ C$, no wind. Vehicle considered is described in Section II–B and [26].

Fig. 9. Optimal SOC profiles on ART LOS AB for different initial SOC values with $T = 20\, ^\circ C$, no wind. Vehicle considered is described in Section II–B and [26].

Fig. 10. Torque thresholds for different initial SOC on ART LOS AB with $T = 20\, ^\circ C$, no wind. Vehicle considered is described in Section II–B and [26].

Fig. 11. Torque thresholds for different environmental conditions on ART LOS AB with 30% initial SOC. Vehicle considered is described in Section II–B and [26].

it can be seen that the transition torque threshold $T_{th}$ level increases with the initial SOC value in order to favor the use of the battery energy. If the initial SOC is high enough (e.g., initial SOC = 95%), the vehicle becomes able to perform the entire driving cycle in pure electric mode. In this case, the hybrid mode is not required anymore and the need for a transition threshold then disappears. However, if at some point of a more demanding driving cycle the battery energy is sufficient but the pure electric mode implies exceeding the electrical motor constraints, the hybrid mode is automatically enforced in order for the engine to assist the electrical motor. This can be done using an extra safety rule that turn the engine on if the electrical motor constraints are violated. Now, if the initial SOC is 95% but the driving cycle is quite longer than ART LOS AB, the battery energy would not be sufficient to perform it all in pure electric mode for the test vehicle described in Section II–B and [26]. In that case, the definition of a transition threshold becomes necessary.

Interestingly, the trip length plays a role equivalent to the initial SOC, when defining the torque threshold function. Increasing the trip length for a fixed initial SOC is equivalent to decreasing the initial SOC for fixed trip distance. Indeed, increasing the trip length will require further assistance of the ICE and then, the transition threshold level will necessarily decrease correspondingly. In conclusion, the transition control law using the introduced torque threshold is viable for different cases of initial SOC.

G. Impact of the Environmental Conditions

The DP algorithm is also applied to various environmental conditions. Changes in wind speed and air temperature are considered. The torque thresholds inferred from DP load points in these conditions are illustrated in Fig. 11. A vehicle facing a head wind must increase its traction torque as opposed to a vehicle in a no-wind case. Consequently, the torque/speed load points will be shifted up and the transition threshold adapts to this new situation. Temperature can also change the optimal behavior. As the air density is inversely proportional to temperature, cold temperature increases drag, and consequently the wheel traction torque, $T_{req}$. It can be concluded that the introduced torque threshold can be used as transition control law also in case of environmental conditions variations.

H. Generalized Form of the Hybrid/Pure Electric Transition Law

As explained in the introduction, if the driving cycle of a future route can be forecasted by any method, it is important
for the EMS to be able to compute optimal power split control parameters in order to minimize the fuel consumption on the route to come. Using DP for this purpose is usually difficult as it requires several hours on a usual desk computer. In this section, a mathematical description of the previously described torque threshold is addressed by introducing transition control parameters. This mathematical description will be used to develop a fast-Genetic Algorithm (GA) able to sensibly decrease the optimization time of the transition control parameters on any given driving cycle.

Observing the shape of the torque threshold of the last section, it can be expressed above 800 rpm as a speed-dependent function approximated by a second-degree polynomial of the form

\[ T_{th} = aN_{EM}^2 - bN_{EM} + c \]  

(5)

where the polynomial coefficients \(a\), \(b\) and \(c\) are the control parameters of the transition control law and \(N_{EM}\) the electric motor speed. This transition control law is easy to use on a real-time controller. Since the electrical motor is always connected to the rear wheel (with the gear configuration illustrated in Section II–B), the electrical motor speed \(N_{EM}\) is proportional to the wheel speed. In the controller, the wheel speed \(N_w\) and required torque \(T_{req}\) (only dependent on the driving cycle) are first measured and then translated into the electrical motor torque/speed plan by a simple division. If the obtained working point is over the threshold, the hybrid mode is enforced. If the obtained working point is under the threshold, the electric mode is enforced.

### III. GENETIC ALGORITHM FOR TRANSITION PARAMETER OPTIMIZATION

The Genetic Algorithm (GA) will proceed to a search of the optimal values of \(a\), \(b\), \(c\) of (5) that will minimize the vehicle fuel consumption over a predefined driving cycle. The GA implemented considers that each individual, within the GA population, is a set of transition parameters \(a\), \(b\), \(c\). To each individual corresponds a torque threshold, \(T_{th}\), given by (5). For each time step of a given driving cycle, the wheel torque \(T_{req}\) is computed. If the corresponding torque \(T_{EM}\) for the EM is below \(T_{th}\) then the ICE is turned off. On the contrary, when the EM torque \(T_{EM}\) required for operation in pure-EV is above the threshold, the ICE is turned on and a hybrid power split ratio is applied.

At this stage, it should be noted that whenever the ICE is turned on, the GA applies a power split so that \(T_{ICE}\) is set to the value of maximum ICE efficiency. This is done according to the control law defined in Section II–D. This rule is applied whatever driving cycle is processed by the GA.

The fitness function (6) of the GA computes the weighted sum of the total fuel consumption, \(f_{uel_{cost}}\), and the SOC deviation for a specified set of transition parameters \(a\), \(b\) and \(c\) (individual).

\[ f_{fit}(a, b, c) = f_{uel_{cost}} + \lambda |SOC_f - SOC_{tar}| \]  

(6)

The final SOC obtained when applying a defined set of transition parameters \(a\), \(b\) and \(c\) on the considered driving cycle is noted \(SOC_f\) while the targeted final SOC is noted \(SOC_{tar}\).

The parameter \(\lambda\) must be set to balance the two objectives of the optimization. If it is set too low, the GA optimization leads to the final SOC which is too low compared to the targeted one, implying possible battery damage. If it is set too high, the SOC deviation will take too much importance in the optimization process and the fuel consumption will be poorly minimized. It has been found that a simple iterative search is sufficient to find a suitable value of \(\lambda\). In practice, it has been observed that one value of \(\lambda\) can be used for all the driving cycles of Table II while keeping near-optimality.

At each time step, the GA works on improving two subpopulations of 50 individuals towards an optimal solution. During the improvement step, the GA makes a random selection with higher probability for the best individuals to be selected. It then makes random-heuristic crossovers among the selected individuals, called parents, with the hope to obtain better individuals, called children. The algorithm also creates children by uniform random mutations in order to prevent early convergence and ensure better exploration of the search space.

In order to validate the reliability of the proposed GA, it is applied to the 10 defined driving cycles, along with their various initial SOC and environmental conditions. At first, it has been observed that the GA performs global optimization in 2 to 3 minutes when DP required several hours. It can then be considered that the objective of reducing the control parameters optimization time is reached. The optimized values of \(a\), \(b\) and \(c\) computed by the GA, along with their corresponding conditions, are listed in Table III.

These parameters can lead to torque threshold shapes that can be different from the shapes obtained by simple observations of the DP results. Several factors can explain these differences. First, the ICE cannot perform instantaneous start-stops and the resulting necessary delay represents an extra constraint that DP does not consider because of its backward nature. The GA takes

<table>
<thead>
<tr>
<th>Initial SOC</th>
<th>(a) [Nm/rpm²]</th>
<th>(b) [Nm/rpm]</th>
<th>(c) [Nm]</th>
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<tbody>
<tr>
<td>30%</td>
<td>-7.64 × 10⁻⁷</td>
<td>8.01 × 10⁻³</td>
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<tr>
<td>ART LOS AB</td>
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<td>6.29 × 10⁻²</td>
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<td>ART LOS CD</td>
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<tr>
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<td>1.27 × 10⁻¹</td>
<td>281.6</td>
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<td>1.33 × 10⁻¹</td>
<td>330.2</td>
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<tr>
<td>FW LOS AC</td>
<td>3.99 × 10⁻⁵</td>
<td>4.40 × 10⁻²</td>
<td>128.6</td>
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<td>FW LOS E</td>
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<td>6.44 × 10⁻²</td>
<td>136.4</td>
</tr>
<tr>
<td>FW LOS F</td>
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<td>1.16 × 10⁻¹</td>
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<tr>
<td>LOCAL</td>
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<td>8.88 × 10⁻³</td>
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<table>
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<tr>
<th>Initial SOC</th>
<th>(a) [Nm/rpm²]</th>
<th>(b) [Nm/rpm]</th>
<th>(c) [Nm]</th>
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<td>40%</td>
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</tr>
<tr>
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<td>-2.65 × 10⁻²</td>
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<tr>
<td>ART LOS AB</td>
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<tr>
<td>Initial SOC</td>
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<td></td>
</tr>
<tr>
<td>30%</td>
<td>2.09 × 10⁻⁶</td>
<td>2.47 × 10⁻²</td>
<td>81.4</td>
</tr>
<tr>
<td>T = -20 °C</td>
<td>1.96 × 10⁻⁶</td>
<td>1.92 × 10⁻²</td>
<td>67.7</td>
</tr>
</tbody>
</table>

**TABLE III**

**Optimized Control Parameters Obtained by Genetic Algorithm on the Test Vehicle of Section II–B and [26]**
the timing delay constraint into account. Second, the GA uses the control law established in Section II–D for the power split decision in hybrid mode but this law can slightly differ from the optimal behavior given by DP. Third, choosing a torque threshold by pure observation of the DP results as done in Section II can be inaccurate. Finally, the optimal torque threshold can be non-unique.

IV. FUEL CONSUMPTION COMPARISON BETWEEN DP AND GA

As explained before, a close-to-optimal fast algorithm is much preferable than an optimal-lengthy one when it comes to a vehicle EMS. Since the rapidity of the GA optimization has been assessed in the previous section, this section concentrates on verifying the near-optimality of the proposed GA.

Fig. 12 illustrates the evolution of the battery SOC resulting from 1) DP and 2) the threshold-based solution described in Section II, which is defined by (5) and where \( a, b \) and \( c \) have been optimized with the GA algorithm of Section III. For both DP and GA, the ART LOS AB driving cycle was considered with an initial SOC of 50%, no wind, and an ambient temperature of 20 °C. The SOC evolution obtained with GA is close to the optimal SOC profile obtained with DP (a maximum difference inferior to 1.9%).

Table IV gives the increase between the achievable minimum fuel consumption obtained with DP and the consumption obtained with GA for various driving and environmental conditions. The difference in fuel consumption is 2.8% (average) and less than 4.9% in all cases. It increases for lower speed profiles such as ART LOS EF and FW LOS G. GA performs the optimization of the parameters \( a, b \) and \( c \) while considering the control law of the PSR in hybrid mode that maximizes the ICE efficiency.

It can be concluded from Table IV that this simple control law can be used for each driving cycle without modification or adaptation and still lead to near-minimal consumption when used in combination with the GA.

It is also relevant to compare the proposed GA optimization to a basic rule-based control strategy. It has been chosen to compare the performances of both approaches on the UDDS driving cycle that represents an urban route with frequent stops. This driving cycle is represented in Appendix B and is one of the federal test procedure (FTP) driving cycles used in the US.

The basic rule-based strategy that was chosen for comparison is based on the power follower strategy which has been used to develop the control strategy of the Toyota Prius and the Honda Insight [3], [28], [29]. This strategy was originally developed for HEVs but can be easily adapted to PHEVs. During a trip, the adapted rule-based control strategy depletes the battery by making the vehicle work like an EV until the low SOC threshold is reached. Then the vehicle works like an HEV and performs charge sustaining operation using the power follower approach.

The power-follower strategy remains popular and ensures good reliability and robustness. Since the rules of this control strategy are mainly based on the SOC feedback, the battery is always operated safely. This method is also simple to implement because it only needs information on the present state of the vehicle (SOC, vehicle speed and \( T_{req} \)). However, such simplicity will lead to a non-optimal power split control. During the charge sustaining operation, the adapted rule-based control strategy uses the ICE as the primary source of mechanical power with the EM providing additional power when the required power cannot be fulfilled by the ICE alone. The electrical motor can also work in regenerative mode when the ICE provides more power than needed to the wheel. The adapted power follower strategy is composed of three main modes that are listed below.

1) Power-Follower Strategy Electric Mode

In this mode, the ICE is shut off and the electrical motor works alone. This mode is chosen for low speed operation and also in vehicle braking operation. In this latter case, the motor is used as a generator to replenish the battery.
Fig. 13. SOC comparison on UDDS between DP, GA, and the rule-based strategy. Vehicle considered is described in Section II–B and [26].

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Computational need</th>
<th>Need the knowledge of the future driving cycle</th>
<th>Fuel consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>DP</td>
<td>Heavy (hours)</td>
<td>Yes</td>
<td>1.89 L/100 km</td>
</tr>
<tr>
<td>GA</td>
<td>Light (2 min.)</td>
<td>Yes</td>
<td>1.94 L/100 km</td>
</tr>
<tr>
<td>Rule-based</td>
<td>Light (seconds)</td>
<td>No</td>
<td>2.58 L/100 km</td>
</tr>
</tbody>
</table>

2) Power-Follower Strategy Hybrid mode

In this mode, the ICE is operated at its optimum efficiency and the electrical motor works either in regenerative or motor mode, depending on the required power. If the SOC drops too low, the ICE may work above the optimum efficiency load point in order to charge the battery pack.

3) Power-Follower Strategy ICE Mode

In this mode, the ICE works alone. It is selected when the state of charge is considered too low. It prevents any further charging of the battery pack. It should be noted that in the present case, this is not a real ICE-only mode because the architecture does not incorporate a clutch to mechanically decouple the EM. Instead, the EM is driven at zero torque.

For the three considered strategies (DP, GA, Rule-Based), the targeted final SOC is 30% and the initial SOC is 80% so that a significant contribution of the ICE is needed to complete the driving cycle. After the simulation of the three strategies, the obtained SOC profiles have been compared and the result is illustrated in Fig. 13. For the basic rule-based strategy, the transition from the EV operation to the charge sustaining operation can distinctly be seen. DP prefers a more progressive charge depleting strategy by using the hybrid mode before the SOC reaches 30%. Furthermore, by using the GA-based control strategy, a SOC evolution like the one obtained with the optimal DP strategy is obtained. Table V shows the characteristics of the three strategies in terms of computation requirement, required information and fuel consumption. As opposed to the rule-based strategy, it can be seen that the GA-based strategy leads to a fuel consumption that is close to optimality.

The reason why the rule-based strategy performances are worse than for the GA is that the parameters of the rule-based strategy are fixed and chosen arbitrarily, while the GA optimizes the control strategy parameters based on the driving cycle, initial SOC and environmental conditions. The chosen rule-based strategy could lead to better fuel consumption if some further optimization process were carried out to tune the control parameters. This optimization process is the main focus of this paper since, through the presented results, it is stressed that 1) the nature of the parameters of a rule-based strategy should be carefully chosen through DP optimization; and that 2) the parameters values should be optimized based on the driving cycle, the initial SOC, and the environmental conditions.

For the UDDS driving cycle, the GA computed the set of optimal transition parameters $a$, $b$, $c$: $a = -1.65 \times 10^{-6}$ Nm/rpm$^2$, $b = 7.23 \times 10^{-3}$ Nm/rpm and $c = 77.9$ Nm. However, even though the computation requirement is low and the optimized parameters can be used in real-time, the GA still needs the knowledge of the future driving cycle.

V. EXPERIMENTAL RESULTS

The real-world test drives were realized by means of the prototype illustrated in Fig. 14, which specifications are given in Table I and [26]. The energy storage is a 2.5 kWh Li-ion battery. A specific route of distance 11 km was selected, which was run twice: first attempt was to measure the speed profile, wheel speed and torque as a function of time. The corresponding speed and required torque profiles are illustrated in Fig. 15.

Before attempting the second drive test, the recorded results from the drive test were passed through the both DP and GA optimization, which provided the $a$, $b$, $c$ values defined by (7) and where the $T_{th}$ function is also plotted with such values when (7) is inserted into (5), as presented in Fig. 16.

\[
a = 5.8 \times 10^{-6}, \quad b = 7.9 \times 10^{-1}, \quad c = 89 \tag{7}
\]

It was discussed earlier how critical were the initial and final SOC in the process of determining the most suitable power split function. In this experiment and optimization, an initial SOC of
with the GA algorithm implemented, whereas the DP theoretical optimization with 52% of SOC as a final value forecasted a 2.31 l/100 km. The difference between these two theoretical and experimental values is 11%, where 4% can be explained by the GA optimization itself and the remaining 7% can be explained by model uncertainties, road condition, traffic difference, wind conditions, pilot position on the vehicle. But in general, we consider this method of using GA algorithm for determining the power split strategy as validated by the experimental results obtained.

VI. Conclusion

In this paper, a GA has been developed for the optimization of the power split control parameters used in the energy management of a PHEV. The developed GA can perform global optimization in much less time than a DP optimization requires, with maximum error in SOC of 1.9% and average increase in fuel consumption of 2.8%. Furthermore, it provides control parameter values leading to near-minimal fuel consumption on a wide variety of driving cycles and these parameters are easily usable in a real-time control law. These features are essential to overcome some important obstacles toward a future real-time implementation. However, as a global optimization tool, the GA still needs the previous knowledge of the driving cycle so, at this stage, a direct real-time implementation is still to be investigated. Future works will focus on associating the developed GA optimization tool with driving cycle prediction or machine learning in order to develop a fully autonomous energy management system and confirm the usefulness of the GA optimization by experiment.

An experimental test drive has been run with the control laws proposed in this paper, which are determined by a Genetic Algorithm optimization. The experiment gives a 2.57 l/100 km fuel consumption on a 11 km route and was validated by a DP optimization of the modeled vehicle on the same driving pattern and distance. It is concluded that the proposed strategy is suitable for the control adaptation to a given route.

APPENDIX

A. Vehicle Gearbox Ratios Used in The Simulations

<table>
<thead>
<tr>
<th>Gear Name</th>
<th>Gear Ratio</th>
<th>Gear Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Value</td>
<td>Symbol</td>
</tr>
<tr>
<td>Primary gear</td>
<td>$i_{pr}$</td>
<td>75/52</td>
</tr>
<tr>
<td>Secondary gear</td>
<td>$i_{sec}$</td>
<td>1</td>
</tr>
<tr>
<td>Motor gear</td>
<td>$i_{mg}$</td>
<td>50/28</td>
</tr>
<tr>
<td>Final gear</td>
<td>$i_{fin}$</td>
<td>79/28</td>
</tr>
<tr>
<td>Gearbox</td>
<td>$i_{gb}$</td>
<td>1st: 32/13 (k = 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd: 28/16 (k = 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3rd: 29/21 (k = 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4th: 27/22 (k = 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5th: 26/23 (k = 5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6th: 25/24 (k = 6)</td>
</tr>
</tbody>
</table>
B. Driving Cycles (FACILITY and UDDS)

![Driving Cycles](image)

REFERENCES


Nicolas Denis was born in France, in 1987. He received the M. Eng. degree and the French Engineer Diploma degree in electrical engineering from the École Nationale Supérieure d’Electricité et de Mécanique de Nancy, Nancy, France, in 2010, and the Ph.D. degree in electrical engineering from the University of Sherbrooke, Sherbrooke, QC, Canada, in 2014. He was a Postdoctoral Fellow and Commissioned Scientist in the Toyota Technological Institute, Nagoya, Japan, from 2014 to 2017. He is currently an Engineer at Challenergy Inc., Tokyo, Japan, where he is developing optimal control to maximize the generated electrical energy of a wind turbine using the Magnus effect.

Maxime R. Dubois (M’04) was born in Alma, QC, Canada. He received the B. Sc. degree in electrical engineering from the Université Laval, Quebec City, QC, Canada, in 1991, and the Ph.D. degree (cum laude) from Delft University of Technology, Delft, The Netherlands, in 2004.

Since 2004, he has been Faculty Member in two Canadian Universities. Between 2004 and 2011, he was with the Université Laval. Since 2011, he has been an Associate Professor in the Department of Electrical Engineering, University of Sherbrooke, Sherbrooke, QC, Canada, where he holds the Canadian Research Chair position in Efficient Electric Vehicles with Hybridized Energy Storage Systems. His research interests include the areas of electric vehicles, hybridized energy storage systems, energy management, and rotating electrical machines. He was the General Co-Chair and the Technical Program Committee Co-Chair of the 2014 IEEE Vehicle Power and Propulsion Conference, as well as the Award Committee Member for the 2015 and 2016 IEEE Vehicle Power and Propulsion Conferences. He is the Technical Program Committee Co-Chair of the 2017 IEEE Vehicle Power and Propulsion Conference. He was a Guest Editor for the Special Issue of IET Electrical Systems in Transportation on Energy Storage and Electric Power Sub-Systems for Advanced Vehicles. He is a Guest Editor for the Special Issue of IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY on ELECTRIC POWER-TRAINS FOR FUTURE VEHICLES.

João Pedro F. Trovão (S’08–M’13–SM’17) was born in Coimbra, Portugal, in 1975. He received the M.Sc. degree and the Ph.D. degree in electrical engineering from the University of Coimbra, Coimbra, Portugal, in 2004 and 2013, respectively. From 2000 to 2014, he was a Teaching Assistant and an Assistant Professor in the Polytechnic Institute of Coimbra–Coimbra Institute of Engineering (IPC–ISEC), Portugal. Since 2014, he has been a Professor in the Department of Electrical Engineering and Computer Engineering, University of Sherbrooke, Sherbrooke, QC, Canada, where he is the founder of Eocycle Technologies Inc., and Founding Professor of the companies AddEnergie Tech. and IngeniArts Tech. He holds six international patents and acts as a consulting engineer on a regular basis. His fields of interest are electrical machines and power electronics applied to the field of wind energy, energy storage and electric vehicles. He is a registered Engineer, and has published 75 papers in scientific journals and conference proceedings. He was Technical Program Chair of the 2015 VPPC conference in Montréal, Canada and a Guest Editor for the Special Issue of IET Electrical Systems in Transportation on Design, Modeling and Control of Electric Vehicles.

Alain Desrochers received the Engineering degree from École Polytechnique de Montréal, Montréal, QC, Canada, the M.Sc. degree from the University of California in Los Angeles, Los Angeles, CA, USA, and the Ph.D. from École Centrale Paris, Gif-sur-Yvette, France. He is the NSERC Chair in Design for Aluminum, whose mandate is to foster the training of design engineers with specific knowledge pertaining to the strategic use of aluminum in products. Prior to this, from 2005 to 2009, he was an Associate Dean Resource in the Engineering Faculty, University of Sherbrooke, and holder of the Bombardier Chair in modeling and design of mechanical systems and complex structures from 1999 to 2004. Since 2010 onward, he was principal investigator in two major projects addressing the design of an electrical roadster (11.6 M$) and the structural lightning of recreational vehicle frames (3.35 M$). His current research interests are in product design and aluminum.