Magnet Shaping for Minimal Magnet Volume in Machines

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***Abstract*—An expression for the no-load flux linkage in permanent-magnet (PM) machines is derived. From this expression, a method is proposed to increase the no-load flux linkage generated in a stator winding per volume of PM material. The method is suitable for machines with short pole pitch and is applied to a transverse-flux machine with single-sided surface-mounted PM. The result is an increase of 37% of the no-load flux linkage per volume of PM, compared with the case of rectangular PM covering 100% of the pole pitch. An increase of 15% is also obtained, if compared with the common practice of using PM covering 80% of the pole pitch.**

***Index Terms*—Permanent-magnet machines, permanent-magnet shaping, transverse-flux permanent-magnet machines.**

I. Introduction

RARE-EARTH permanent magnets (PMs) are expensive. The specific cost of Nd–Fe–B and Sm–Co material is many times higher than that of steel and copper. The optimization of PM volume is investigated, by evaluating the contribution of each PM volume element to the no-load magnetic flux in a machine.

The rotor of a PM electrical machine comprises an alternate arrangement of north poles and south poles formed by PMs, which create an alternating flux in the stator winding. In this paper, the no-load flux linkage in the stator winding is expressed as a mathematical function integrated over the PM volume. This gives each volume element inside the PM a contribution to the total no-load flux linkage. A method is presented that eliminates the PM elements with low contribution to the flux generated in the stator winding. Higher value of flux linkage per cubic meter of PM material can then be obtained.

The method is applied to a transverse-flux PM machine. A pole pitch of 1 cm is used for that purpose, which exhibits a high content of fringing field in the air gap and in the magnets. The magnet shape resulting from this method is compared to the more conventional rectangular magnet layout, which has a clearance of 0.1 to 0.3 times the pole pitch between each magnet.

II. Contribution of Magnet Elements to the No-Load Flux

In this section, an expression for the stator flux linkage $\lambda_{\rm PM}$ created by the PM at no-load is derived. This expression is different from the conventional expression given by (1)

$$\lambda_{\rm PM} = \iint_{S_{\rm coil}} \vec{B}_{\rm PM} \cdot d\,\vec{s}_{\rm coil} \tag{1}$$

where $\boldsymbol{B}_{\rm PM}$ is the flux density created by the PM, and $S_{\rm coil}$ is a surface bounded by the coil conductors. In (1), the coil geometry is considered, not the PM geometry. The new expression for $\lambda_{\rm PM}$ is derived, by injecting a current i in the coil. The product $i\lambda$ is written as

$$i\lambda = \left[\iint_{S_{\rm cond}} \vec{j} \cdot d\,\vec{s}_{\rm cond}\right]\left[\iint_{S_{\rm coil}} \vec{B} \cdot d\,\vec{s}_{\rm coil}\right] \tag{2}$$

where $\boldsymbol{j}$ is the current density inside the conductors forming the coil, $S_{\rm cond}$ is the conductor cross section, $\boldsymbol{B}$ is the total flux density created in the coil by both the current i and the PM. If $\boldsymbol{B}$ is replaced by the curl of the magnetic vector potential $\boldsymbol{A}$

$$\vec{B} = \nabla \times \vec{A} \tag{3}$$

we can apply Stoke's theorem, and obtain

$$i\lambda = \left[\iint_{S_{\rm cond}} \vec{j} \cdot d\,\vec{s}_{\rm cond}\right]\left[\oint_{l_{\rm coil}} \vec{A} \cdot d\vec{l}_{\rm coil}\right] \tag{4}$$

which can be expressed by (5), if the variation of $\boldsymbol{A}$ throughout the conductor cross section is negligible

$$i\lambda = \iiint_{V_{\rm cond}} \vec{j} \cdot \vec{A}\, dv \tag{5}$$

where $V_{\rm cond}$ is the total volume of the conductors linking the coil. Using (3) and (6), and using the method applied in [1], (5) can be rewritten into (7)

$$\vec{j} = \nabla \times \vec{H} \tag{6}$$

$$i\lambda = \iiint_{V_{\rm universe}} \vec{B} \cdot \vec{H}\, dv \tag{7}$$

where $V_{\rm universe}$ is the total volume of universe. From (7), we can separate $\lambda, \boldsymbol{B}$, and $\boldsymbol{H}$ into the components $\lambda_{\rm PM}, \boldsymbol{B}_{\rm PM}, \boldsymbol{H}_{\rm PM}$ created by the PM, and $\lambda_a, \boldsymbol{B}_a, \boldsymbol{H}_a$ created by the stator current. If steel and PMs are considered as linear material, we can write

$$i(\lambda_{\rm PM} + \lambda_a) = \iiint_{V_{\rm universe}} (\vec{B}_{\rm PM} + \vec{B}_a) \cdot (\vec{H}_{\rm PM} + \vec{H}_a)\, dv. \tag{8}$$

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For the case considered here, Maxwell's equations give

$$\nabla \cdot \vec{B}_{\mathrm{PM}} = \nabla \cdot \vec{B}_a = \nabla \times \vec{H}_{\mathrm{PM}} = 0. \tag{9}$$

The integral overall universe of a product of two vectors is zero, if the first vector has a divergence of zero and the second vector has a curl of zero [2]. Using this statement, we rewrite (8)

$$i(\lambda_{\mathrm{PM}} + \lambda_a) = \iiint_{V_{\mathrm{universe}}} \left(\vec{B}_{\mathrm{PM}} + \vec{B}_a\right) \cdot \vec{H}_a \, dv. \tag{10}$$

From (10), we isolate λ_{PM}

$$i\lambda_{\mathrm{PM}} = \iiint_{V_{\mathrm{universe}}} \vec{B}_{\mathrm{PM}} \cdot \vec{H}_a \, dv. \tag{11}$$

If the magnet is assumed to have a constitutive relationship defined by (12)

$$\vec{B}_{\mathrm{PM}} = \mu_0 \vec{H}_{\mathrm{PM}} + \vec{B}_r. \tag{12}$$

As shown in [3], (11) and (12) can be combined, and written as

$$i\lambda_{\mathrm{PM}} = \iiint_{V_{\mathrm{universe}}} \vec{B}_a \cdot \vec{H}_{\mathrm{PM}} \, dv + \iiint_{V_{\mathrm{PM}}} \vec{H}_a \cdot \vec{B}_r \, dv. \tag{13}$$

The first term is zero, because the divergence of $\boldsymbol{B}_a$ is zero, and the curl of $\boldsymbol{H}_{\mathrm{PM}}$ is zero. This leaves the following result:

$$\lambda_{\mathrm{PM}} = \iiint_{V_{\mathrm{PM}}} \frac{\vec{H}_a}{i} \cdot \vec{B}_r \, dv. \tag{14}$$

Equation (14) has the advantage of expressing the no-load flux linkage in the stator coil as a volume integral performed on the PM. In (14), each volume element dv inside the PM gives a certain contribution to the total flux linkage λ_{PM}.

A method of PM volume optimization can be implemented, with the use of (14). This can be done with a finite-element analysis (FEA) software, by replacing the PM by vacuum in the FEA model. A current i is injected in the stator coil, and the resulting field $\boldsymbol{H}_a$ is obtained in the volume previously occupied by PM. The dot product of $\boldsymbol{H}_a$ with $\boldsymbol{B}_r$ must be performed and divided by i. The local contributions are then obtained, and the PM volume elements with low contribution can be eliminated from the total volume.

Inspection of (14) readily indicates that such a method will give equal contributions inside the PM, if the field $\boldsymbol{H}_a$ is constant in direction and amplitude inside the PM volume. This method will be more helpful in the case of machines with very short pole pitch, where two- and three-dimensional (2-D and 3-D) fringing fields are present. The contribution inside the PM will then vary for each volume element.

III. Magnet Shaping in Short-Pole-Pitch Machines

A. Description of the TFPM SSSM Machine

The method described in the last section is applied to a transverse-flux permanent-magnet (TFPM) machine, with single-sided surface magnets (SSSMs) [4], shown in Fig. 1.

The main characteristic of TFPM machines is the independence between the amount of stator-created magnetomotive force (MMF) and the rotor-created no-load flux linkage. In other words, the window in which the conductors are placed can be made many times wider than the pole pitch, which is not the case with conventional brushless dc PM machines. This allows the pole pitch in the TFPM SSSM of Fig. 1 to be made very small, while still keeping a large winding window area. Small pole pitches increase the rate of change of the no-load flux linkage, which, in turn, increase the no-load voltage.

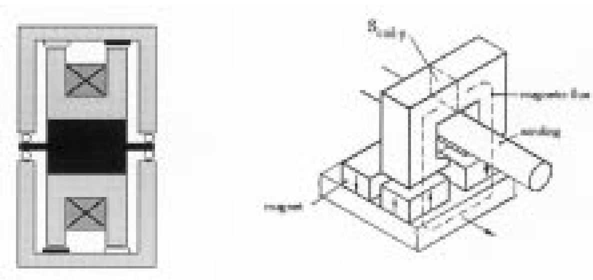

Fig. 1. Cross-sectional and 3-D views of a TFPM SSSM machine.

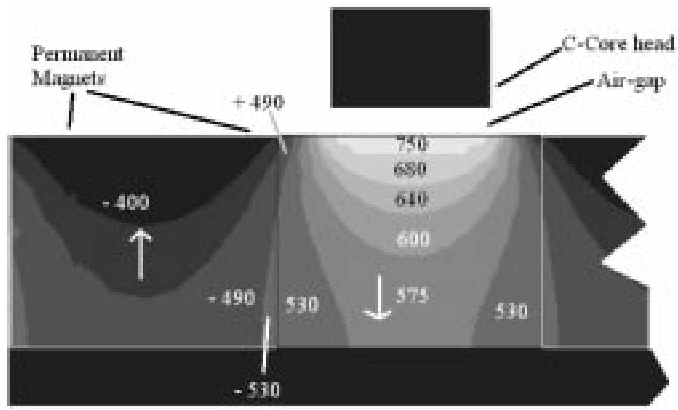

Fig. 2. Dot product of $\boldsymbol{H}_a$ and $\boldsymbol{B}_r$ inside the volume of adjacent block Nd–Fe–B magnets. MMF injected in the stator is $ni = 10$ ampere-turns (A-turns). Width of PM = 10 mm. Distance between PM = 0 mm. Height of PM = 8 mm. Total area of PM per magnet = 80 mm^2. $B_r = 1.23$ T.

The TFPM SSSM machine is composed of C-cores in the stator, mounted on the interior part of the machine. The stator winding is made of one single coil wound on top of the inner part of the C-cores. The PMs are placed on the inner part of the outer ring, and face the C-core heads. For this study, a pole pitch of 1 cm is used, which is also the value used in [4].

B. Derivation of Magnet Shapes With Increased Contribution

A current $i = 10$ amperes is injected in the stator coil of the TFPM SSSM machine, and three cases of PM configurations are investigated: PM with no clearance between them, PM with a clearance of 0.2 times the pole pitch between them, and PM with areas of lower $\boldsymbol{H}_a \cdot \boldsymbol{B}_r$ removed. The three PM shapes with the corresponding plots of $\boldsymbol{H}_a \cdot \boldsymbol{B}_r$ are shown in Figs. 2–4. It must be noted that the same magnet height is used for the three shapes. Further optimization of magnet material utilization could be achieved if the magnet height is allowed to vary, which is not done here.

The no-load flux linkage λ_{PM} is computed for the three shapes and displayed in Table I. λ_{PM} is calculated by using both the conventional (1) and the new expressions (14) for flux

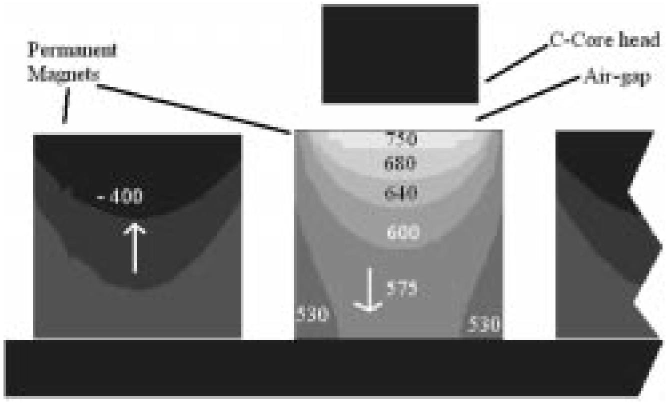

Fig. 3. Dot product of $\boldsymbol{H}_a$ and $\boldsymbol{B}_r$ inside the volume of rectangular Nd–Fe–B magnets. MMF injected in the stator is $ni = 10$ A-turns. Width of PM = 8 mm. Distance between PM = 2 mm. Height of PM = 8 mm. Total area of PM per magnet = 64 mm^2. $B_r = 1.23$ T.

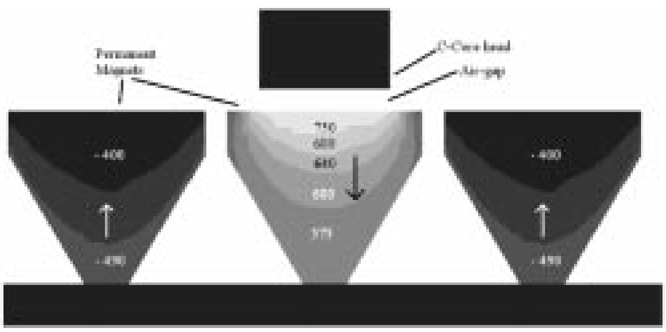

Fig. 4. Dot product of $\boldsymbol{H}_a$ and $\boldsymbol{B}_r$ inside the volume of the PM of V-shaped Nd–Fe–B magnets. MMF injected in the stator is $ni = 10$ A-turns. Distance between PM in the top part = 1 mm. Width of PM in the bottom part = 2 mm. Distance between PM in the bottom part = 8 mm. Total height of PM = 8 mm. Height of the top (rectangular) part of the PM = 2 mm. Total area of PM per magnet = 51 mm^2. $B_r = 1.23$ T.

linkage. The expression of $\lambda_{\rm PM}$ calculated from (14) shows an error of 8% with the conventional expression. This is explained by a relative recoil permeability of 1.09 used for the PM in the FEA model, where (14) assumed a value of 1. The PM area is also shown in Table I, and the ratio of $\lambda_{\rm PM}$ over PM volume gives the highest value for the V-shaped PM. The V-shape is 37% more volume effective than the square magnets with no clearance, and 15% more volume effective than the rectangular magnets with clearance.

TABLE I
FEA RESULTS FOR THREE DIFFERENT PM CONFIGURATIONS

PM Shape	λ_{PM} given by $\iint_{S_{coil}} B_{PM} \cdot dS_{coil}$ (Wb/m)	λ_{PM} given by $\iiint_{V_{PM}} \left[\frac{H_a}{i}\right] \cdot B_r\, dv$ (Wb/m)	Error $\Delta\lambda_{PM}$ (%)	PM Surface (mm^2)	λ_{PM}/V_{PM} (Wb/m^3)
Square 100%	0.00215 (μ_{rec}=1.09)	0.00198	8	80	2.7 (μ_{rec}=1.09)
Rectang 80%	0.00205 (μ_{rec}=1.09)	0.00190	7	64	3.2 (μ_{rec}=1.09)
V-shape	0.00188 (μ_{rec}=1.09)	0.00173	8	51	3.7 (μ_{rec}=1.09)

IV. CONCLUSION

An expression for the no-load flux linkage is derived for PM machines, which expresses the no-load flux linkage as a function integrated over the PM volume. From this expression, a method is proposed to increase the no-load flux linkage generated by the PM in the winding per volume of PM material. The method is applied to a machine with a small pole pitch (1 cm), i.e., a transverse-flux machine with SSSM PM. The result is a V-shaped PM, with an increase of 37% of the no-load flux linkage per volume of PM, compared with the case of rectangular PM covering 100% of the pole pitch. An increase of 15% is also obtained, if the V-shaped PM is compared with the common practice of using PM covering 80% of the pole pitch. The no-load flux linkage calculated with (14) shows a maximum error of 8% compared to the conventional expression of no-load flux linkage. This error is attributed to the assumption in (14) of unity recoil permeability for the PM material. The proposed method of computing no-load flux linkage is a valid tool for PM with recoil permeability close to unity.

REFERENCES

[1] J. A. Stratton, *Electromagnetic Theory*. New York: McGraw-Hill, 1941.
[2] W. F. Brown, *Magnetostatic Principles in Ferromagnetism*. Amsterdam, The Netherlands: North-Holland, 1962.
[3] M. R. Dubois, H. Polinder, and J. A. Ferreira, "Contribution of PM volume elements to the no-load voltage in machines," *IEEE Trans. Magn.*, submitted for publication.
[4] M. R. Harris, G. H. Pajooman, and S. M. Abu Sharkh, "Performance and design optimization of electric motors with heteropolar surface magnets and homopolar windings," *Proc. Inst. Elect. Eng.—Elect. Power Applicat.*, vol. 143, pp. 429–436, Nov. 1996.